

Rating Exposure Control Using Bayesian Decision Analysis

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A model is presented for applying Bayesian statistical techniques to the problem of determining, from the usual limited number of exposure measurements, whether the exposure profile for a similar exposure group can be considered a Category 0, 1, 2, 3, or 4 exposure. The categories were adapted from the AIHA exposure category scheme and refer to (0) negligible or trivial exposure (i.e., the true $X_{0.95} \leq 1\%OEL$), (1) highly controlled (i.e., $X_{0.95} \leq 10\%OEL$), (2) well controlled (i.e., $X_{0.95} \leq 50\%OEL$), (3) controlled (i.e., $X_{0.95} \leq 100\%OEL$), or (4) poorly controlled (i.e., $X_{0.95} > 100\%OEL$) exposures. Unlike conventional statistical methods applied to exposure data, Bayesian statistical techniques can be adapted to explicitly take into account professional judgment or other sources of information. The analysis output consists of a distribution (i.e., set) of decision probabilities: e.g., 1%, 80%, 12%, 5%, and 2% probability that the exposure profile is a Category 0, 1, 2, 3, or 4 exposure. By inspection of these decision probabilities, rather than the often difficult to interpret point estimates (e.g., the sample 95th percentile exposure) and confidence intervals, a risk manager can be better positioned to arrive at an effective (i.e., correct) and efficient decision. Bayesian decision methods are based on the concepts of prior, likelihood, and posterior distributions of decision probabilities. The prior decision distribution represents what an industrial hygienist knows about this type of operation, using professional judgment; company, industry, or trade organization experience; historical or surrogate exposure data; or exposure modeling predictions. The likelihood decision distribution represents the decision probabilities based on an analysis of only the current data. The posterior decision distribution is derived by mathematically combining the functions underlying the prior and likelihood decision distributions, and represents the final decision probabilities. Advantages of Bayesian decision analysis include: (a) decision probabilities are easier to understand by risk managers and employees; (b) prior data, professional judgment, or modeling information can be objectively incorporated into the decision-making process; (c) decisions can be made with greater certainty; (d) the decision analysis can be constrained to a more realistic "parameter space" (i.e., the range of plausible values for the true geometric mean and geometric standard deviation); and (e) fewer measurements are necessary whenever the prior distribution is well defined and the process is fairly stable. Furthermore, Bayesian decision analysis provides an obvious feedback mechanism that can be used by an industrial hygienist to improve professional judgment. For example, if the likelihood decision distribution is inconsistent with the prior decision distribution then it is likely

that either a significant process change has occurred or the industrial hygienist's initial judgment was incorrect. In either case, the industrial hygienist should readjust his judgment regarding this operation.

Keywords Bayesian statistics, exposure assessment, exposure rating

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INTRODUCTION

Industrial hygiene has often been described as both an art and a science. The art component often consists of the application of professional judgment in determining whether occupational exposures are acceptable, relative to some occupational exposure limit (OEL). Professional judgment is needed because we are often compelled to make decisions based on limited information. For example, consider a well-defined exposure group with 50 workers. Assuming roughly 250 work days per year, the population of exposures each year consists of 12,500 worker-days. If only 6 to 10 measurements per year can be collected, the plant industrial hygienist (IH) is forced to make a decision regarding the acceptability of a distribution of exposures for 12500 worker-days using a statistical sample of no more than 0.08% of the population. Because the sample size is small, the resulting statistical confidence intervals around any calculated statistics are often large. Yet, a decision has to be made. For example, consider the following scenario:

A process with a volatile component is situated in a large open room with high ceilings and considerable dilution ventilation. For a particular group of workers, three full-shift personal exposure measurements were collected: 0.20, 0.05, and 0.10 ppm. All were considerably less than the exposure limit of 1 ppm. The point estimate of the group 95th percentile

TABLE I. AIHA Exposure Categorization Scheme

Exposure Category ^A	Rule-of-Thumb Description ^B	Qualitative Description	Recommended Statistical Interpretation ^C	Notes
0	Exposures are trivial to nonexistent—employees have little to no exposure, with little to no inhalation contact.	Exposures, if they occur, infrequently exceed 1% of the OEL.	$X_{0.95} \leq 0.01 \times \text{OEL}$	1
1	Exposures are <i>highly controlled</i> —employees have minimal exposure, with little to no inhalation contact.	Exposures infrequently exceed 10% of the OEL.	$0.01 \times \text{OEL} < X_{0.95} \leq 0.1 \times \text{OEL}$	2
2	Exposures are <i>well controlled</i> —employees have frequent contact at low concentrations and rare contact at high concentrations.	Exposures infrequently exceed 50% of the OEL and rarely exceed the OEL.	$0.1 \times \text{OEL} < X_{0.95} \leq 0.5 \times \text{OEL}$	2, 3, 4
3	Exposures are <i>controlled</i> —employees have frequent contact at low concentrations and infrequent contact at high concentrations.	Exposures infrequently exceed the OEL.	$0.5 \times \text{OEL} < X_{0.95} \leq \text{OEL}$	2, 4
4	Exposures are <i>poorly controlled</i> —employees often have contact at high or very high concentrations.	Exposures frequently exceed the OEL.	$X_{0.95} > \text{OEL}$	4

^A An exposure category can be assigned to a SEG whenever the true 95th percentile exposure ($X_{0.95}$) falls within the specified range.

^B The “Rule-of-thumb” descriptions were based on similar descriptions published by the AIHA.⁽²⁾

^C $X_{0.95}$ = the true group 95th percentile exposure.

Notes:

1—Category 0 was added to distinguish between highly-controlled exposures and situations where exposures are either nonexistent or trivially low. It was included in the 1991 AIHA rating scheme.⁽²⁾ 2—“Infrequently” refers to an event that occurs no more than 5% of the time. 3—“Rarely” refers to an event that occurs no more than 1% of the time. 4—“High concentrations” are defined as concentrations that exceed the TWA OEL.

was 0.31 ppm, which suggests that exposures for the group tend to be a Category 2 exposure (using the American Industrial Hygiene Association [AIHA] exposure categorization scheme, Table I). However, the 95%UCL (upper confidence level) for the group 95th percentile exposure was 20.2 ppm, or more than 20 times the limit, suggesting that there was considerable statistical uncertainty in the point estimate. The dozens of full-shift measurements collected from similar processes in similar circumstances were always well below the limit. Even though only three measurements were collected and the 95%UCL($X_{0.95}$) was considerably greater than the limit, the IH concluded that this operation was well controlled. Routine surveillance monitoring was recommended; no follow-up verification survey should be necessary.

Several observations are possible. First, today’s state-of-the-art guidance suggests that one should calculate an upper percentile exposure, such as the 95th percentile, as well as its 95% upper confidence limit, and compare both with the time-weighted average (TWA) OEL before making a final decision.⁽¹⁾ Given that only three measurements were collected, the 95% upper confidence limit for the 95th percentile exposure often exceeds the exposure limit, in this case it was exceeded by a factor of 20. Assuming that a “well-controlled” work environment is one where 95% of the exposures are less than 50% of the exposure limit, consideration of the statistics alone would suggest that such a decision could not possibly

be made with high confidence. In our scenario, however, the IH supplemented his consideration of the limited exposure data and the statistical analysis with personal experience and professional judgment.

Is there a way to quantify this professional judgment? Can professional judgment be objectively introduced into the decision-making process? This article introduces a methodology that addresses these questions, which we will call Bayesian Decision Analysis (BDA).

BACKGROUND

Exposure Profiles and Exposure Categories

A frequent objective when collecting exposure data is to classify the exposure profile, or distribution of exposures, for a similar exposure group (SEG) into one of five exposure categories: 0, 1, 2, 3, or 4, corresponding to trivial (or very low) exposure, highly controlled, well controlled, controlled, and poorly controlled exposures. Table I lists each of these categories, or substance specific “control band,” along with the statistical description suggested by the AIHA^(1,2) for each exposure category. (Note that the control part of the category description refers to the *effective* level of control; it does not imply that engineering or other controls have actually been applied. For example, natural ventilation may be sufficient for a particular operation to be rated highly controlled, even though

TABLE II. Typical Actions or Controls That Result for Each Final Rating

Final Rating	Action or Control
0	No action
1	General or chemical specific hazard communication
2	Chemical specific hazard communication
3	Chemical specific hazard communication Exposure surveillance Medical surveillance Work practice evaluation
4	Chemical specific hazard communication Exposure surveillance Medical surveillance Work practice evaluation Respiratory protection Engineering controls
4+	Immediate engineering controls or process shutdown Validate that respiratory protection is appropriate

the employer does not actively attempt to reduce exposures.) Assigning the exposure profile to the correct exposure category can be critically important in that specific actions or controls are performed or implemented for each category. Table II contains a listing of typical actions and controls.

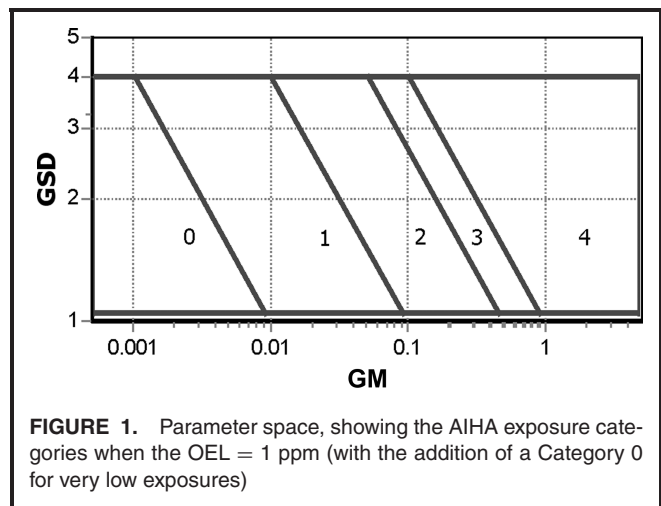
Using the AIHA exposure categorization scheme, an acceptable exposure group is one where the true group 95th percentile exposure (for a reasonably homogeneous group) is less than the single shift exposure limit, or L_{TWA} .⁽¹⁾ Consequently, an unacceptable exposure group is one where the true 95th percentile exceeds the limit. An acceptable exposure profile can be further categorized as a Category 0, 1, 2, or 3 exposure profile (see Table I).

Using the previous example, where the limit is 1 ppm, we can create a map of the exposure rating parameter space (Figure 1) that shows the five exposure categories. For example, it follows from the statistical descriptions in Table I that any combination of true group geometric mean (G) and group geometric standard deviation (D) that results in a 95th percentile that is between 1% and 10% of the limit can be considered a Category 1 exposure profile. Similar reasoning applies to the other exposure categories.

Mathematically, the potential group G and D values extend to infinity. In reality, there are physical constraints for both variables. It is obvious that our exposure rating map is bounded by our selections of probable minimum and maximum values for G and D: G_{min} , G_{max} , D_{min} , and D_{max} . Following the convention in Bayesian statistics, we will call this exposure map the “parameter space.”

Conventional Approach

With conventional statistics the focus is strictly the data (e.g., exposure measurements), calculation of one or more



“compliance” statistics, followed by the calculation of confidence intervals that reflect the degree of statistical uncertainty in the sample statistics. For example, using the previous example we can calculate the following statistics:

descriptive statistics: sample geometric mean = 0.1 ppm
sample geometric standard deviation = 2.0
compliance statistics: sample 95th percentile ($X_{0.95}$) = 0.31 ppm
95%UCL = 20.2 ppm

The 95% upper confidence limit (95%UCL) is used as an indicator of our degree of uncertainty in the point estimate and in this case suggests that there is considerable uncertainty. This leads to our collective dilemma—reaching a defensible decision with limited data. For example, consider the previous scenario where the point estimate of the 95th percentile was less than 50% of the TWA OEL, suggesting a decision that the exposure profile merits a Category 2 rating, yet the 95%UCL was more than 20 times the exposure limit. What should be our decision? Should we focus on the point estimate and conclude that the exposure profile is a Category 2, or should we look at the UCL and conclude that the exposure profile may be a Category 4.

Absent any prior information or experience, we could decide that the exposure profile for this exposure group appears to be a Category 2 exposure profile, but because the 95%UCL($X_{0.95}$) greatly exceeds the limit we would not have high confidence in our conclusion. Consequently, we should either collect additional data to narrow the confidence interval, or schedule a follow-up survey within the near future to verify our provisional decision. See Mulhausen and Damiano⁽¹⁾ for further guidance regarding verification surveys.

However, if we had favorable experience with this type of operation we would be tempted to conclude that in this instance the exposure profile was a Category 2. But how would we express our confidence in a decision that now includes a measure of professional judgment? Even though traditional statistics offers no formal mechanism for

objectively factoring professional judgment or prior data into this particular decision-making process, the company IH went ahead and intuitively and qualitatively factored into his decision his prior experience with similar operations. However, Bayesian statistics allows us to objectively factor in prior data or professional judgment into the decision-making process.

THE BAYESIAN MODEL

The Bayesian approach to statistics and decision making is based on the equation developed by the Reverend David Bayes and published in 1763.^(3,4) We adapted this equation to the situation where we are trying to determine which of k lognormal exposure profiles best describes our data:

$$P(\ln G_i, \ln D_i | \text{data}) = \frac{P(\text{data} | \ln G_i, \ln D_i) \cdot P(\ln G_i, \ln D_i)}{\sum_{i=1}^k [P(\text{data} | \ln G_i, \ln D_i) \cdot P(\ln G_i, \ln D_i)]} \quad (1)$$

In this equation and throughout this article, data refers to a log-transformed dataset of exposure measurements $y = \{y_1, y_2, \dots, y_n\}$ where $y = \ln(x)$ for a particular exposure group (or an individual worker). The G_i and D_i pair refers to the i th lognormal exposure profile from which the data may have been collected. Equation 1 can be read as the probability of the i th exposure profile, given our data, equals the probability of observing this set of data, given the i th exposure profile, times the probability of the i th exposure profile. The denominator is a normalization factor that ensures that the probabilities across all k exposure profiles sum to one.

As it stands, Eq. 1 cannot be applied to determining the probability that our data come from a specific exposure category, as each consists of an infinite number of possible combinations of G and D . Later we will further modify Eq. 1 so that Bayesian methods can be applied to industrial hygiene decision making.

The Prior, Likelihood, and Posterior

There are three components to Eq. 1: the prior distribution of decision probabilities ($P(\ln G_i, \ln D_i)$), the *likelihood* distribution of decision probabilities ($P(\text{data} | \ln G_i, \ln D_i)$), and the *posterior* distribution of decision probabilities ($P(\ln G_i, \ln D_i | \text{data})$).

The Prior Distribution

The $P(\ln G_i, \ln D_i)$ quantity represents the *a priori* probability that the true exposure profile is the i th exposure profile. Prior decision distributions come in two varieties: a noninformative and an informative prior decision distribution. The noninformative prior (or a uniform or flat prior) decision distribution is used to represent the situation where we have little to no prior knowledge or expectations regarding this particular process, in which case we assume that each of the exposure profiles being considered are equally likely. For example, if we must choose between two specific exposure profiles the noninformative prior decision probability would be 0.5 for each. In the Bayesian literature, a noninformative prior distribution can be

used to represent “complete ignorance” regarding a particular set of choices. Use of a noninformative prior might be appropriate when we are evaluating an operation that we have never before seen or characterized, or an existing operation that has changed substantially since the last evaluation. However, we would argue that because of our experience and knowledge we are rarely completely ignorant regarding any exposure scenario.

The informative prior decision distribution is used as a means of expressing quantitatively our experience or expectations regarding an exposure scenario. (Note that the prior, likelihood, and posterior decision distributions, as they are used in this article, do not refer to distributions of exposures or any other physical quantity. They simply refer to the distribution of the decision probabilities among the five exposure categories.) An informative prior can be developed, for example, from previous surveys at this or similar operations, physical/chemical modeling calculations, or the professional judgment of a panel of experienced professionals.

The Likelihood Distribution

The likelihood distribution function represents the relative probability of observing this set of data, given a specific combination of G and D . For lognormally distributed occupational exposure data, this probability is proportional to the likelihood function:

$$P(\text{data} | \ln G_i, \ln D_i) = K \cdot \prod_{j=1}^n \text{pdf}(y_j | \ln G_i, \ln D_i) \quad (2)$$

where K is a proportionality constant. (The proportionality constant is not important as it cancels out when Eq. 2 is used later in Eqs. 3, 4, and 5.)

For occupational exposure data, the likelihood function is simply the product of the lognormal probability density function (pdf) calculated across all n values in the current dataset. (It is usually convenient to calculate the likelihood function by summing the natural logs of the probability density functions for all values and then exponentiating the sum.) The pdf is calculated as follows:

$$\text{pdf}(y | \ln G_i, \ln D_i) = \frac{1}{\ln D_i \sqrt{2\pi}} \cdot \exp\left(\frac{-(y - \ln G_i)^2}{2(\ln D_i)^2}\right)$$

Because Eq. 2 yields only a relative probability, the following equation is used to calculate the *likelihood* probability estimate:

$$P(\text{data} | \ln G_i, \ln D_i) = \frac{\prod_{j=1}^n \text{pdf}(y_j | \ln G_i, \ln D_i)}{\sum_{i=1}^k \left[\prod_{j=1}^n \text{pdf}(y_j | \ln G_i, \ln D_i) \right]} \quad (3)$$

The numerator consists of the likelihood function calculated using the current dataset and a specific combination of groups G and D . The denominator sums the likelihood function over the k exposure profiles, so that the likelihood probabilities sum to one.

The Posterior Distribution

The posterior distribution probability $P(G_i, D_i | \text{data})$ is calculated using Eq. 1 and represents the mathematical combination of the prior distribution and the likelihood distribution, and reflects our final decision probability regarding the i th exposure profile.

Bayesian Model Applied to the AIHA Exposure Category Scenario

As written, Eq. 1 can be applied only to the situation where there are a fixed number of possible exposure profiles. It must be modified to apply to the problem of determining which population of exposure profiles (i.e., exposure category) is most likely, given our data and prior experience. The following equation can be used to determine the posterior probability of the i th exposure category:

$$P(\text{Pop}_i | \text{data}) = \frac{\int_{\ln G'}^{\ln G_{\max}} \int_{\ln D'}^{\ln D_{\max}} [P(\text{data} | \ln G, \ln D) \cdot P(\text{Pop}_i)] d(\ln G) d(\ln D)}{\int_{\ln G_{\min}}^{\ln G_{\max}} \int_{\ln D_{\min}}^{\ln D_{\max}} [P(\text{data} | \ln G, \ln D) \cdot P(\text{Pop}_i)] d(\ln G) d(\ln D)} \quad (4)$$

Here we are using Pop_i to refer to all combinations of G and D within the i th exposure category. The populations under consideration are the five exposure categories from Table I and Figure 1. Equation 4 can be solved for the i th population or exposure category by calculating the product of the likelihood function (shown in Eq. 2) and probability of that exposure category for all pairs of G and D within our defined parameter space, as represented by G_{\min} , G_{\max} , D_{\min} , and D_{\max} . This will result in a 3-D surface rising above the G and D plane (for example, see Figure 3). (In principle, the exposure categories under consideration should be *exhaustive* and *exclusive*. That is, the exposure categories should represent all possible exposure profiles and there should be no overlap between categories.)

The numerator is the double integral for the likelihood function calculated for all pairs of G and D (represented by G' and D') that fall within the i th exposure category. The denominator represents the total volume under the surface and is necessary so that the resulting category probabilities sum to one. The calculations can be complicated and require the use of 3-D integration techniques to determine the decision probability for each exposure category.

The Prior Decision Distribution

The $P(\text{Pop}_i)$ quantity represents the *a priori* probability that the exposure profile for this type of process falls within the i th exposure category. As before, prior decision distributions come in two varieties: noninformative and informative. The noninformative prior distribution is used to represent the situation where we have little to no prior knowledge or expectations regarding this particular process, in which case we assume that each of the populations are equally likely:

Category	$P(\text{Pop}_i)$
0—trivial	0.20
1—highly controlled	0.20
2—well controlled	0.20
3—controlled	0.20
4—poorly controlled	0.20

An informative prior decision distribution can be used as a means of expressing quantitatively our experience or expectations regarding an exposure scenario. For example, let us assume that across a corporation the fraction of time that similar operations have been rated Categories 0, 1, 2, 3, and 4 were:

Category	$P(\text{Pop}_i)$
0—trivial	0.05
1—highly controlled	0.20
2—well controlled	0.50
3—controlled	0.20
4—poorly controlled	0.05

In this scenario, before collecting any exposure measurements, we have quantitatively expressed our belief that the exposure profile for the operation in question is most likely a Category 2.

As seen in Figure 2, an informative prior decision distribution can be plotted as a Prior Decision Function across the parameter space. There are other more mathematically rigorous methods for describing the prior that are discussed later, but for ease of use we chose to interpret the prior decision distribution as a set of weightings. Note that all points (i.e., G and D pairs) within each exposure category receive the same prior probability, or weighting. According to Bayesian statistics, this is an improper prior in that it is not a true probability density function: the volume under the entire function is not equal to one and the relative volume for

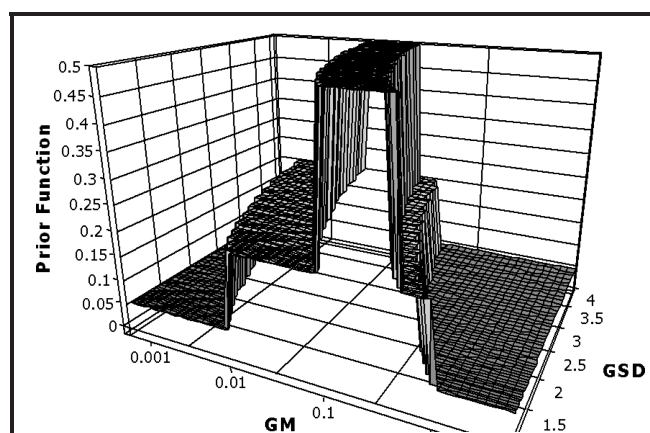


FIGURE 2. An example of an informative Prior Decision Distribution, expressed as a Prior Function spread throughout parameter space

each category is not necessarily equal to the assigned category probability. However, since a correction factor can easily be calculated that will result in a volume of one without changing the relative weightings, the improper prior decision function as we have plotted can be used without modification.⁽⁵⁾

The Likelihood Decision Distribution

Equation 5 allows us to calculate the probability that the data come from each exposure category (without consideration of our experience or expectations):

$$P(\text{data} | \text{Pop}_i) = \frac{\int_{\ln G_{\min}}^{\ln G_{\max}} \int_{\ln D_{\min}}^{\ln D_{\max}} P(\text{data} | \ln G, \ln D) d(\ln G) d(\ln D)}{\int_{\ln G_{\min}}^{\ln G_{\max}} \int_{\ln D_{\min}}^{\ln D_{\max}} P(\text{data} | \ln G, \ln D) d(\ln G) d(\ln D)} \quad (5)$$

The numerator in Eq. 5 consists of Eq. 2 integrated across all combinations of G and D within each exposure category. The numerator of Eq. 5, which is the Likelihood Decision Function, is calculated and plotted in Figure 3 for the three datapoints in our scenario. The G and D values corresponding to the peak of the Likelihood Decision Function represent the maximum likelihood estimates of the true G and D. Once the Likelihood Decision Function is integrated across each exposure category, and corrected for the total volume (the denominator of Eq. 5 is the normalization factor), the results can be plotted as a Likelihood Decision Chart (the center decision chart in Figure 5).

The Posterior Decision Distribution

The Posterior Decision Function (i.e., the numerator in Eq. 4) is the product of Prior Decision Function (Figure 2) and Likelihood Decision Function (Figure 3), and is plotted in Figure 4. Basically, each point on Figure 2 is multiplied times the corresponding point on Figure 3, which as shown in Figure 4 results in function that is different from the Likelihood Decision Function (Figure 3) due to the influence of the informative prior.

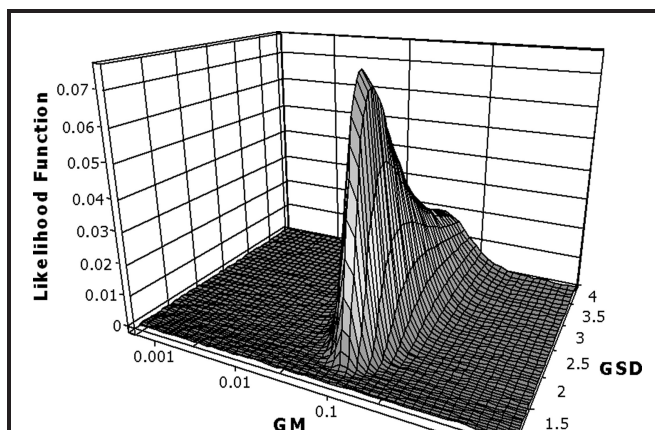


FIGURE 3. The Likelihood Function calculated using Eq. 5 and the example dataset: $x = \{0.20, 0.05, 0.10\}$

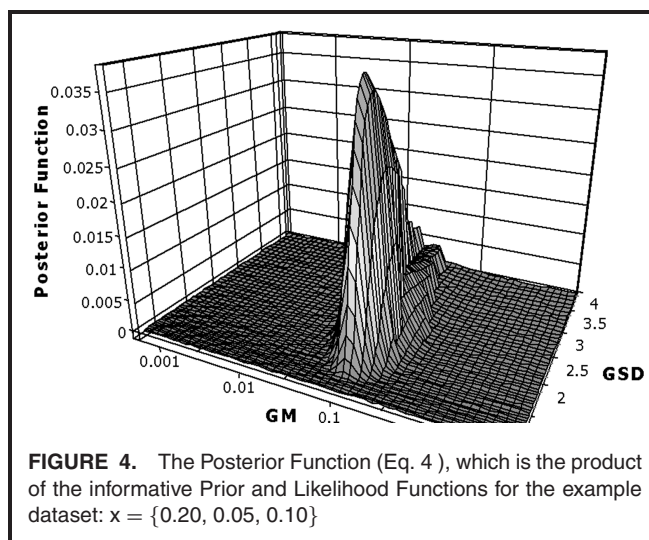


FIGURE 4. The Posterior Function (Eq. 4), which is the product of the informative Prior and Likelihood Functions for the example dataset: $x = \{0.20, 0.05, 0.10\}$

Equation 4 is used to determine the posterior decision probabilities, which are displayed in Figure 6. The posterior decision distribution represents our final decision probabilities, after taking into account the likelihood that the data came from each exposure category and the prior probability that was assigned to each category.

METHODS

To determine the probability that the true exposure profile falls within each of the exposure categories, the Likelihood and Posterior Decision Functions (Figures 3 and 4) must be integrated across each exposure category using Eqs. 5 and 4, respectively. To do this the extent of parameter space must be specified and used as the integration range.

Setting the Integration Boundaries

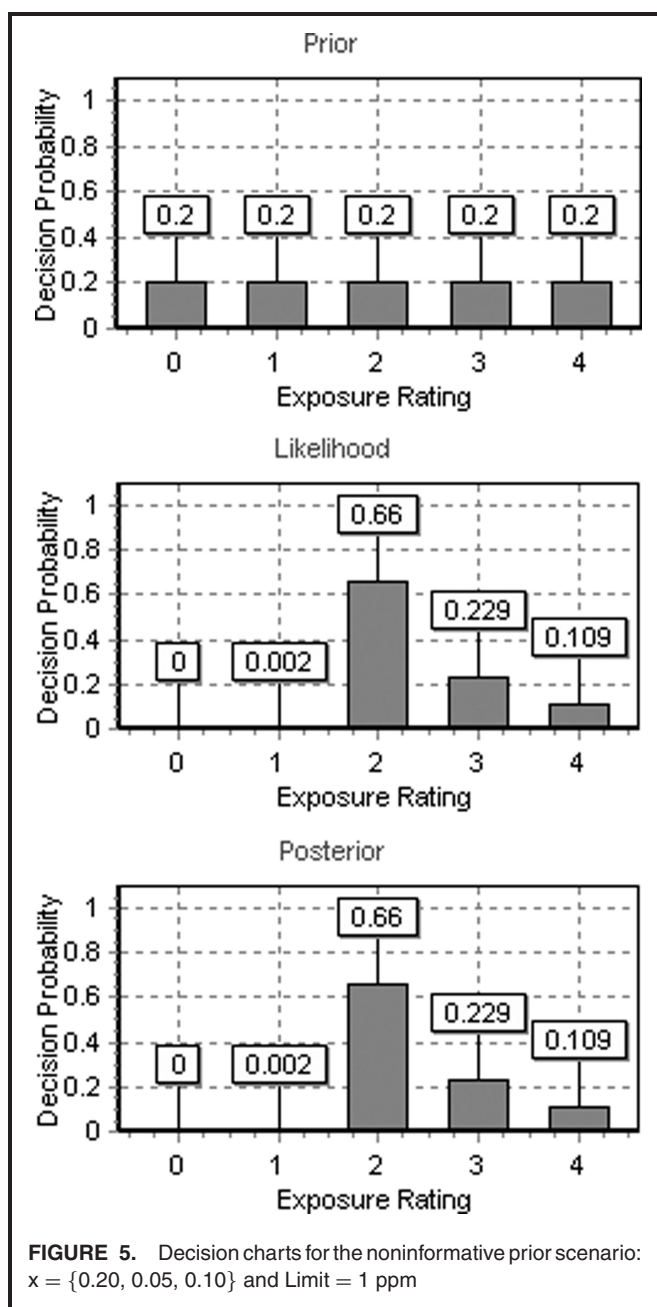
For this article we used the following integration boundaries:

$$\begin{aligned} D_{\min} &= 1.05 & D_{\max} &= 4 \\ G_{\min} &= 0.005 \cdot L_{TWA} & G_{\max} &= 5 \cdot L_{TWA} \end{aligned}$$

D_{\min} and D_{\max} reflect the expectation that a group geometric standard deviation less than 1.05 or greater than 4 is highly unlikely for this particular operation. Similarly, G_{\min} and G_{\max} represent the boundaries of expected or probable true values. These are suggested integration boundaries and can, of course, be modified to fit the situation. For example, say exposure modeling calculations indicate that it is physically impossible for an exposure to exceed a given value. G_{\max} could be set equal to or less than this value. Or, for example, say a group geometric standard deviation greater than 3 had never been observed for this operation. This would warrant setting D_{\max} at 3.

Validation of the Calculations

A computer program was developed to perform the 3D integration calculations of the surfaces represented by Eqs. 4



and 5. We checked our calculations using the open source WinBUGS (Bayesian inference Using Gibbs Sampling) software (version 1.4.1). The WinBUGS software was designed as a generic approach to solving a variety of statistical problems using Bayesian methods. After devising a WinBUGS model equivalent to that presented here (the code is listed in the Appendix), we found that for all datasets tested, the WinBUGS code led to identical or nearly identical likelihood decision probabilities (authors P. L. and S. B.). Slight differences were due to the fact that winBUGS uses a Markov Chain Monte Carlo approach to solving Bayesian problems where the results will vary slightly from run to run. In summary, our calculations were reproduced using a well-recognized program for doing Bayesian calculations.

EXAMPLES

We envision several applications of BDA; it is most useful when the sample size is small, say fewer than 10 measurements. It could be used to help industrial hygienists calibrate their professional judgment and more objectively understand uncertainty. BDA may also provide a means to use modeling predictions^(6,7) of exposure to enhance our interpretation of limited quantitative exposure measurements. These and other hypothetical examples of how BDA could be applied are presented below.

Using a Noninformative Prior

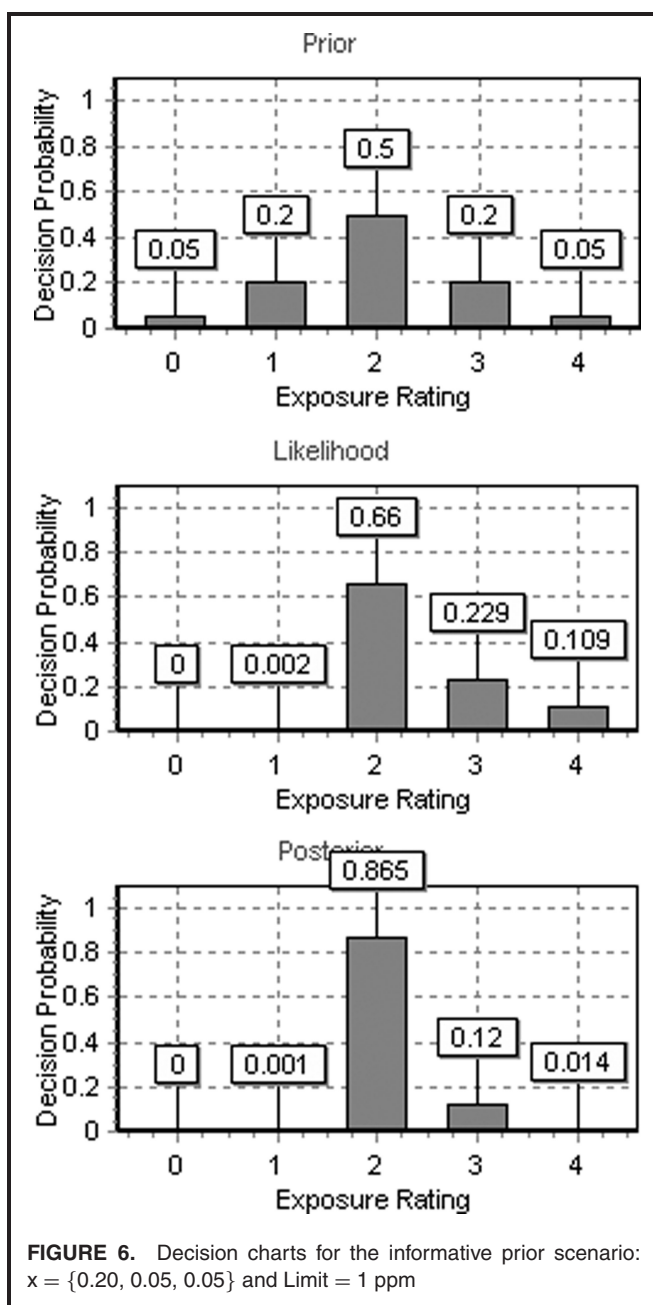
We will use the data from the scenario in the Introduction to demonstrate Bayesian Decision Analysis where a noninformative prior decision distribution is used. The exposure limit L_{TWA} is 1 ppm. Using the integration boundaries discussed above, the BDA results for this example are shown in Figure 5. Using Eqs. 5 and 4 it can be determined that the likelihood and posterior probabilities that the process exposure profile is Category 2 or 3 are 0.660 and 0.229, respectively. There is little probability that the true exposure profile is a Category 0 or 1. However, there is an uncomfortably large 0.109 probability of a Category 4 exposure, which is analogous to saying that given the limited data (and the constraints placed on the analysis in terms of the boundaries of the parameter space) there is a >10% probability that the true SEG 95th percentile exposure is in Category 4.

Notice that the posterior decision distribution in Figure 5 is identical to the likelihood decision distribution. This is because we assumed a noninformative prior. With noninformative priors, final decision probabilities reflect only the analysis of the data. However, even though we assumed a noninformative prior, the BDA analysis gives us a final output that is easier to understand and interpret than the traditional $X_{0.95}$ point estimates and confidence intervals.

Using an Informative Prior

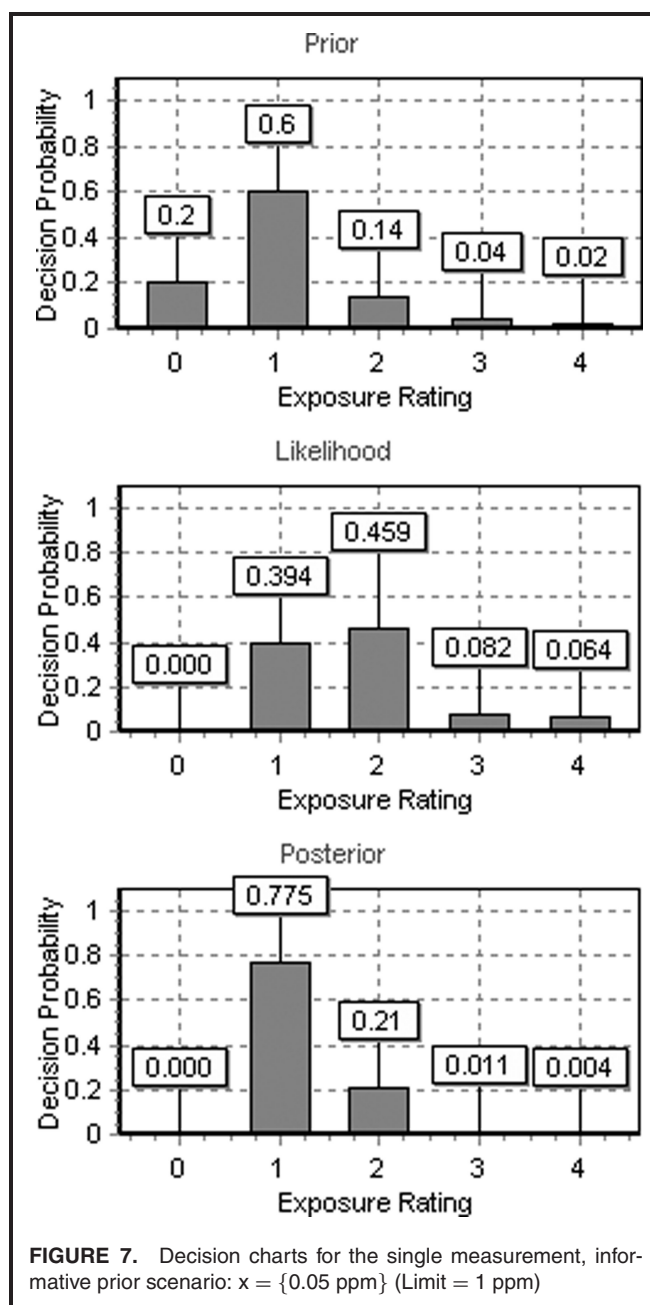
Using the same scenario, let us assume that we have sufficient experience with this type of process that we can construct an informative prior decision distribution prior to the collection of data. Using the informative prior presented earlier and Eqs. 5 and 4, the likelihood and posterior decision probabilities shown in Figure 6 can be calculated.

Because the prior and likelihood decision distributions are consistent (i.e., both predict the same exposure category) the combined posterior probability that the exposure profile is a Category 2 or 3 has increased from 0.660 to 0.865. At the same time the probability of a Category 4 exposure profile has decreased from an uncomfortable 0.109 to a tolerable 0.014. At this point, our IH would be justified in deciding that the exposure profile is most likely a Category 2. When using an informative prior, the final decision is a combination of professional judgment, as reflected in the prior distribution, and an analysis of the current data.



Verifying that Exposures Remain Controlled

Let us assume that a single surveillance measurement was collected from a process that in this and other operations had been rated a Category 1 or 2 for 60% and 14% of the time, respectively, with decreasing percentages for the other exposure categories (see Figure 7). This particular measurement was 5% of the limit. Although only a single measurement was available, most IHs would be comfortable with this situation and conclude that the process is acceptable. With BDA and assuming the above informative prior, the IH would be justified in concluding that there is greater than 98% probability that the exposure profile is no more than a Category 2. A final decision could be reached with high confidence using a single surveillance measurement because the IH was able to



leverage past experience through the use of a prior decision distribution.

Interpretation of Measurements Near the Exposure Limit

Industrial hygienists, whether Occupational Safety and Health Administration (OSHA), Mine Safety and Health Administration, state, local, or corporate, are usually compelled to make decisions based on limited exposure data. Also, historical data are often dated or nonexistent for many SEGs, and it may be unlikely that the industrial hygienist has observed this particular operation. Consequently, one could argue that a noninformative prior distribution would be appropriate in such instances. Even with a noninformative prior, BDA can be used

to better frame the issues that lead to a decision, enhance risk communication with the employer, and help determine whether additional measurements are needed to reach a defensible decision. For example, in Figure 8 we consider the scenario where a single measurement approaches the limit.

Let us assume that a new process was introduced. Due to a limited budget, the plant IH collected only a single measurement that is 95% of the exposure limit. Let us also assume that the IH had no experience with this operation, and that a uniform, noninformative decision distribution was appropriate in this instance. Whereas the single measurement was technically in compliance with the limit, BDA analysis

suggested that this single measurement was most consistent with a Category 4 exposure profile. The IH would be well advised to start investigating, identifying, and controlling the determinants of exposure for the new process.

Calibration of Professional Judgment

Most industrial hygienists develop an initial impression regarding an operation whenever conducting an exposure assessment. For example, the AIHA⁽¹⁾ recommends that operations be given an “initial rating” so that they can be prioritized for quantitative studies (or controlled if the initial rating is a Category 4 and is based on reliable information). A low initial rating, for example, a rating of Category 1 or 2 with high certainty, may not lead to a quantitative study or the implementation of exposure controls.

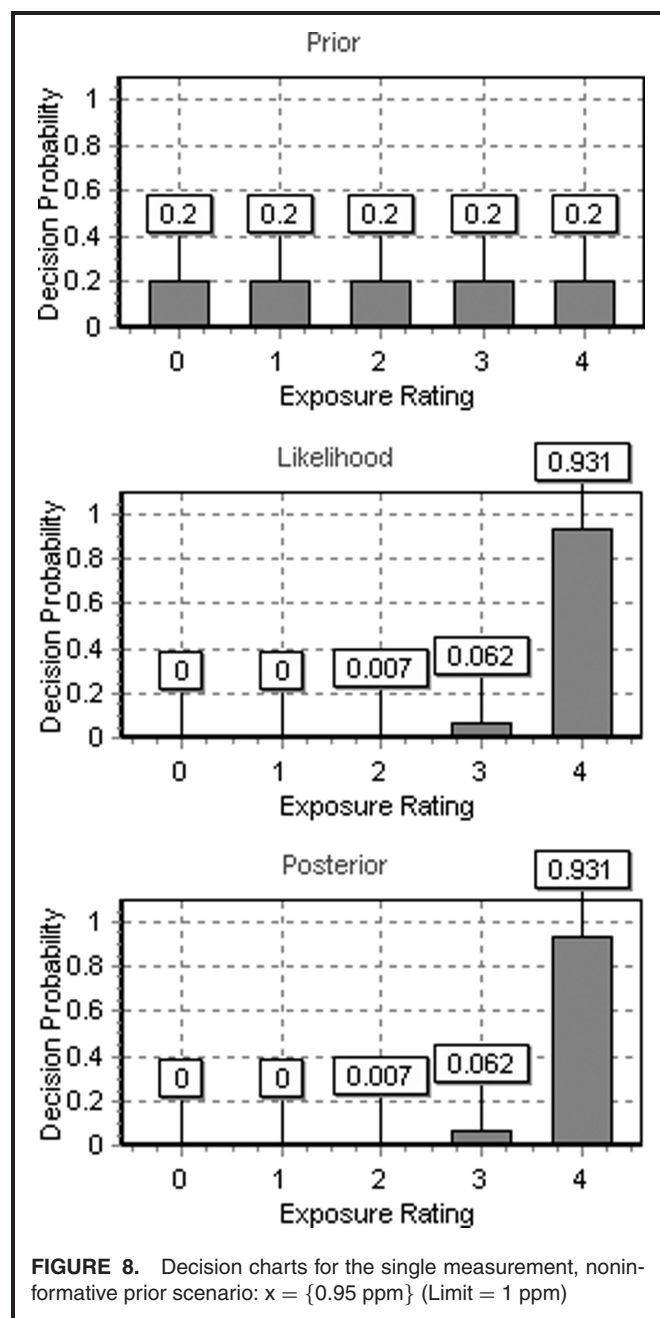
In principle, an IH can be said to be well calibrated whenever the primary category of the prior distribution and the primary category of the likelihood decision distribution agree more often than not. There may be a problem with the calibration of the professional judgment of the IH whenever the prior distribution and likelihood distribution substantially disagree.

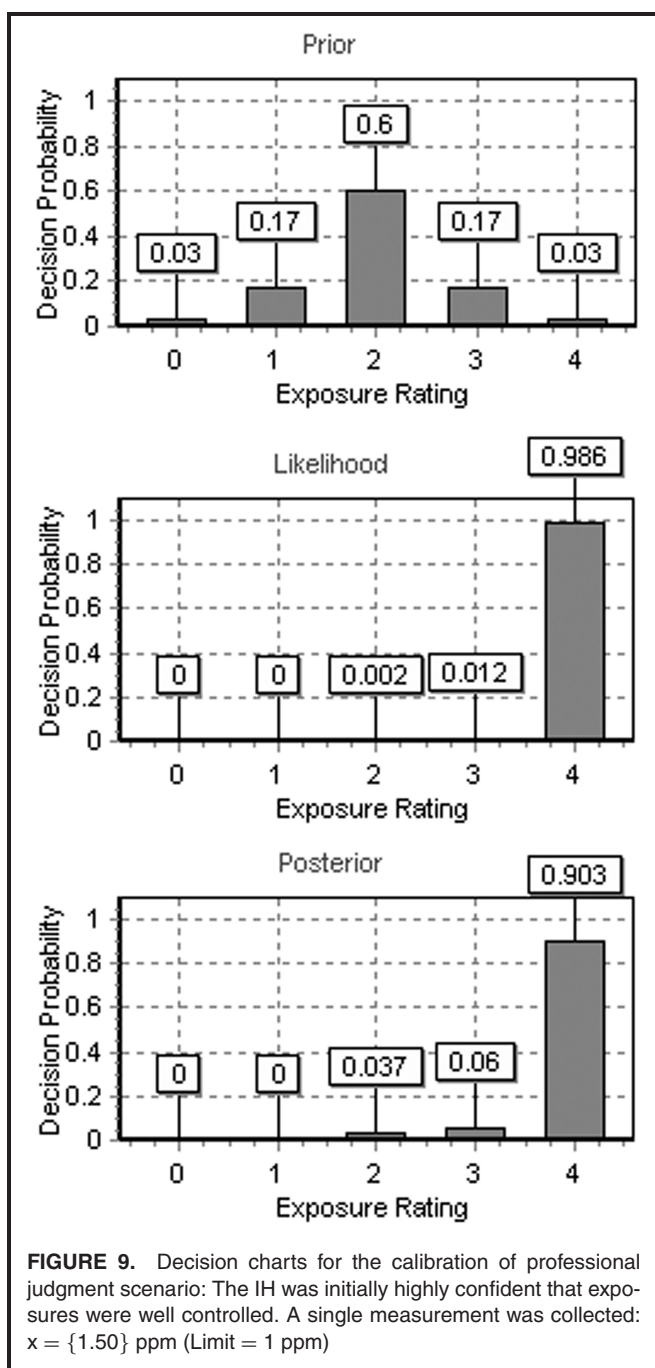
We suggest that BDA could be used as a feedback mechanism to assist IHs to improve or sharpen their professional judgment. In the case where an IH’s initial rating is inconsistent with the likelihood decision distribution, the posterior decision distribution can be misleading. For example, consider the BDA results in Figure 9. Here, a hypothetical IH was initially highly confident that an operation was well controlled (i.e., a Category 2 exposure). A surveillance measurement was collected that exceeded the limit by 50%. The likelihood decision distribution, which reflects a Bayesian analysis of only the data (in this case a single measurement), indicates that true 95th percentile exposure most likely exceeds the OEL and that the SEG exposure profile should be rated a Category 4.

The IH has two options: (1) conclude that the initial rating was wrong and use the likelihood distribution (which is based solely on the exposure data) for reaching a final decision, or (2) collect additional data. If an IH’s initial rating is consistently high or low, then obviously the IH needs to recalibrate his professional judgment.

Application of Exposure Modeling Predictions

Jayjock⁽⁶⁾ suggested that Monte Carlo Simulation techniques could be used to predict exposures. Mulhausen and Damiano⁽¹⁾ and the AIHA⁽⁷⁾ discuss the prediction of exposures based on the physical and chemical attributes of the chemicals and process. Such predictions of exposure could be used to construct a prior decision distribution. Let us assume that our IH used exposure modeling techniques to predict that a 95th percentile exposure greater than 50% of the exposure limit (Limit = 1 ppm) is unlikely. The IH then created a fairly crude prior decision distribution, essentially hedging his bet as to which exposure control rating is most appropriate (see Figure 10). Three measurements were collected $x = \{0.05, 0.10, 0.20\}$. In this case the predicted exposure rating was consistent with the prior decision distribution. Even with this crude prior

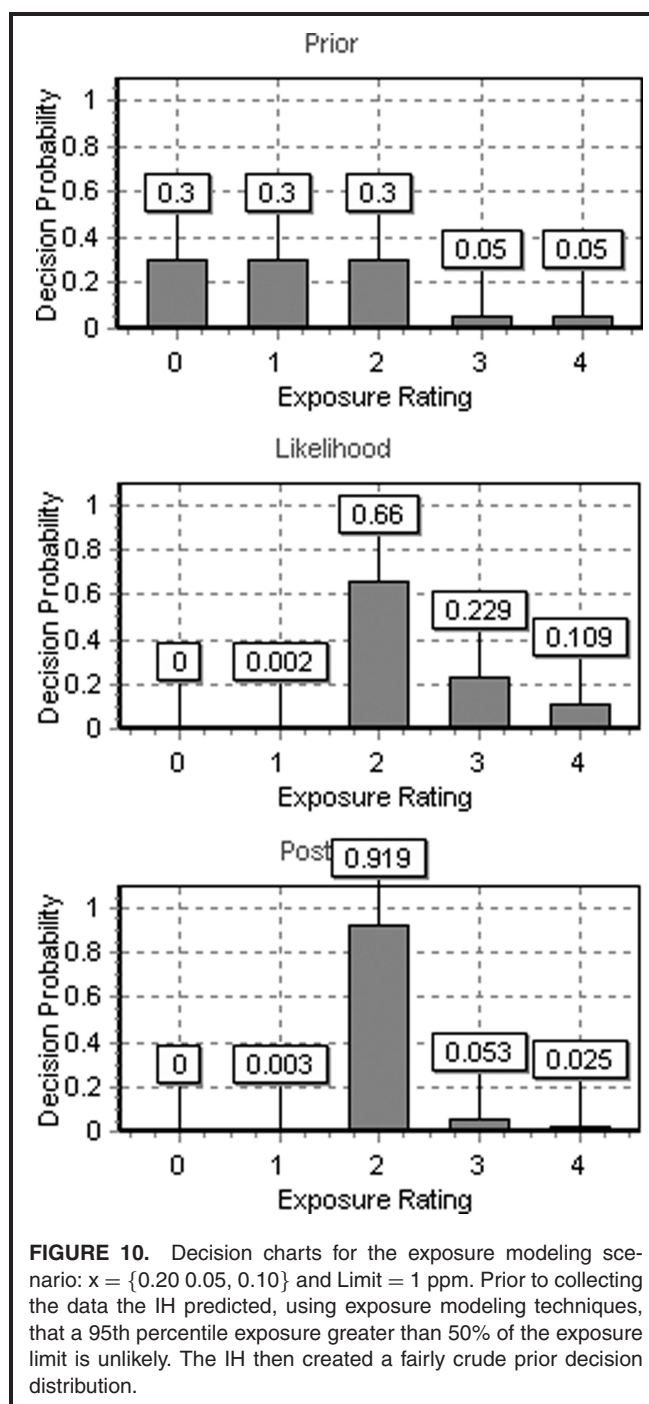




decision distribution, the IH would be able to conclude with 97% certainty that the exposure rating should be a Category 2 or 3.

Selection of Respiratory Protection

Earlier we presented a scenario where a single measurement was collected that approached the exposure limit. Suppose that this caused two additional measurements to be collected resulting in a final dataset of $x = \{0.95, 0.50, 2.0\}$ ppm. Application of BDA would tell us that the exposure profile for this process is most likely a Category 4 exposure, so



the issue now involves the selection of the most appropriate respirator. OSHA guidance in 1910.134 for selecting the appropriate respirator is limited.⁽⁸⁾ BDA can be used to help determine the best choice of respiratory protection by simply replacing the dividing line between the decision categories by respirator assigned protection factors (APF), which are basically multiples of the OEL. The x-axis in the decision charts changes from exposure rating category to APF category; otherwise, the BDA calculations are identical. Figure 11 suggests that in this scenario a respirator with an APF of 10 or 25 would be appropriate.

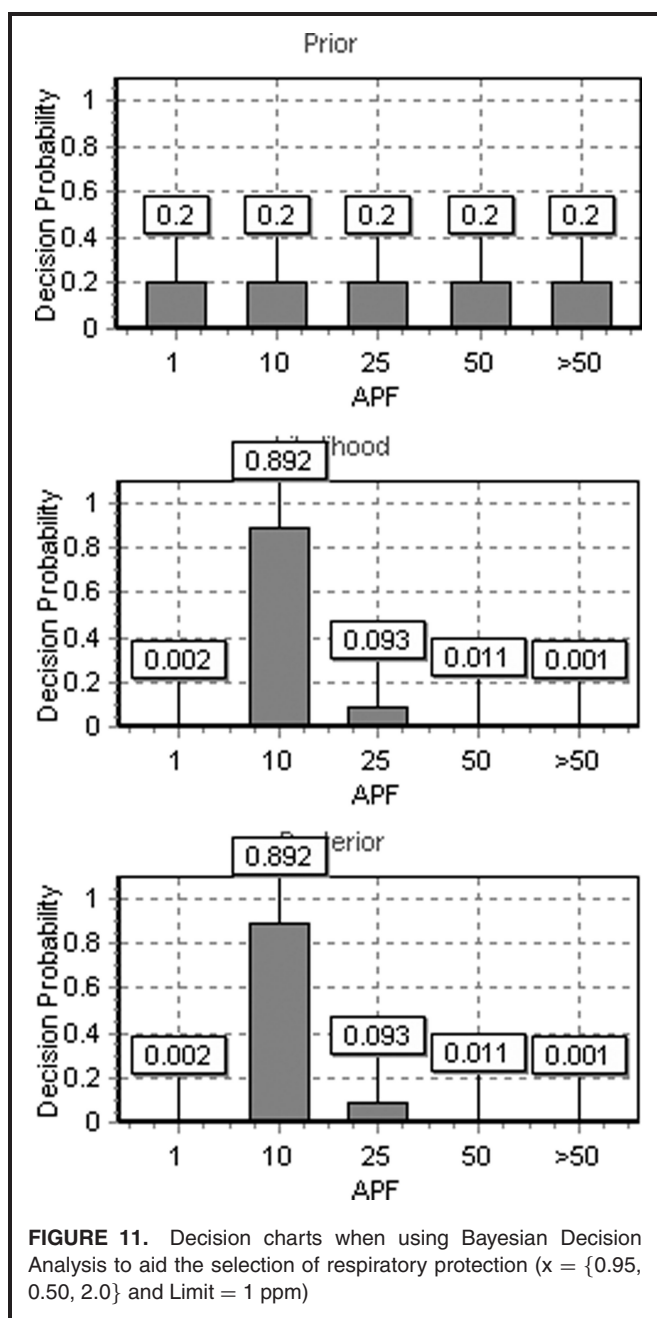


FIGURE 11. Decision charts when using Bayesian Decision Analysis to aid the selection of respiratory protection ($x = \{0.95, 0.50, 2.0\}$ and Limit = 1 ppm)

DISCUSSION

BDA is most useful when the sample size is small. As the sample size increases, the likelihood decision function (e.g., Figure 3) becomes steeper and sharper, and the traditional confidence intervals narrow. At larger sample sizes, say above 10 to 20, the final decisions determined from consideration of conventional statistics and BDA tend to rapidly converge. We suggest that IHs continue to calculate the usual statistics (and confidence intervals whenever the sample size permits) but that BDA be used as an adjunct to assist in interpreting the data and categorizing the process, as well as a means for objectively factoring professional judgment into the decision-making process.

Advantages

Bayesian Decision Analysis provides a means for objectively using our professional judgment in the decision-making process. As illustrated in the introduction, we argue that most industrial hygienists routinely use subjective Bayesian methods when reaching a decision in the presence of limited exposure data. BDA and the use of decision charts permit the IH to bring this mental process out into the open by encouraging the IH to quantify prior experience. The justification or rationale for the prior decision distribution can and should be documented so that the entire decision-making process is transparent and can be reproduced by others.

The analysis can be constrained to a plausible range of exposure profiles. The range of possible exposure profiles does not extend to infinity, or even to very large values (e.g., hundreds of mg/m^3 or tens of thousands of ppm). With conventional statistics, unfortunately, there is a built-in presumption that there are no constraints on the upper confidence limit. This becomes apparent when we calculate, say, the 95% upper confidence limit for the sample 95th percentile for a set of measurements where n is small. For example, consider the following scenario: $n = 2$, sample geometric mean = 0.02 ppm, sample geometric standard deviation = 2.0, and $L_{TWA} = 1$ ppm. The sample 95th percentile is 0.06 ppm, but the 95%UCL is 1.6 million ppm. We know that it is physically impossible for the true 95th percentile to be this great, yet this is the upper range of uncertainty according to the standard statistical methods that we often employ.

With BDA the parameter space can be restricted to a range of plausible values for both the exposure profile geometric mean and geometric standard deviation. We devised what we believe is a reasonable set of default values for G_{\min} , G_{\max} , D_{\min} , and D_{\max} , which can be modified to fit a specific scenario. In principle, a smaller parameter space results in a sharper decision chart.

The focus is on decision making. IHs often calculate statistics and confidence intervals, but in the end the risk managers are usually interested in making a decision that the exposure profile is acceptable or unacceptable, or if it is acceptable, just how acceptable is it: highly controlled, well controlled, or just barely controlled. The point estimate of the 95th percentile assists in determining the most likely control zone for the exposure profile, but a statistical confidence interval around the point estimate tells us little about our confidence in that decision or in alternative decisions. BDA is focused directly at quantifying our confidence in the various decision alternatives.

Exposure modeling could be used to help develop a prior decision distribution. Exposure modeling⁽⁷⁾ based on chemical engineering principles could be used to determine the maximum probable exposure, the most likely exposure, or a range of probable exposures given a range of assumptions. The Monte Carlo simulation approach to modeling⁽⁶⁾ can also be used to determine conservative estimates of the 95th or 99th percentile exposures. Both approaches to exposure modeling could be used to develop and refine a prior decision distribution.

The results tend to match expectations. Most would agree that one or two very low measurements have good predictive value whenever they are collected from a process that historically has been well controlled or highly controlled (i.e., a Category 2 or 1). Bayesian decision charts tend to validate this intuitive feeling and provide a way to quantify our confidence in our decision. At the other end of the spectrum, most IHs are uncomfortable whenever an exposure measurement approaches the exposure limit, especially if the process has never before been evaluated. The particular measurement may be technically in compliance with the exposure limit, but the exposure profile from which it was derived is probably not acceptable. As we saw in Figures 7 and 8, Bayesian decision charts support these “gut feelings.”

BDA can improve communication with management and employees. The decision chart can be easily understood by nonprofessionals. A risk manager’s ability to reach a decision is not hindered by unfamiliar statistics and difficult to interpret confidence intervals.

The interpretation of small datasets is facilitated. Bayesian analysis permits us to calculate decision probabilities for small sample sizes, even for a single measurement. In contrast, conventional confidence intervals on exposure profile statistics tend to be broad whenever the sample size is small, and impossible to calculate for a single measurement.

BDA provides a transparent and more rigorous method for selecting an appropriate type of respirator. If the data indicate that the SEG exposure is most likely a Category 4, the IH must immediately decide on the type of control needed. If engineering controls are not immediately feasible then the proper respirator must be selected. Using the probabilistic approach in BDA, the boundaries between exposure categories can be set at the respirator APFs to facilitate the selection of an appropriate respirator APF.

Disadvantages

The BDA calculations are complex. Implementation of the BDA method will require the user to have programming and mathematical skills sufficient for calculating and integrating Eqs. 4 and 5. For our initial “proof-of-concept” BDA application^(9,10) we implemented BDA using both the common spreadsheet functions and the programming language built into the spreadsheet. We found that the only way to accurately integrate Eqs. 4 and 5 was to use a Monte Carlo simulation approach. More recently we wrote a dedicated program so that Eqs. 4 and 5 could be calculated with sufficient accuracy and Figures 2, 3, and 4 could be plotted.

Application of BDA requires training. Our experience suggests that a user of BDA must be fairly knowledgeable of general statistics and trained in the potential pitfalls of BDA. For example, one of the basic assumptions is that the true exposure profile actually falls within the defined parameter space. The user must check to ensure that the data sample geometric standard deviation is well within the parameter space. If it is not, the parameter space must be enlarged or the dataset analyzed to determine why the sample geometric

standard deviation is so large (e.g., perhaps data from two different SEGs were combined and should be analyzed separately). Below, we further discuss issues related to defining parameter space. Another pitfall is being overconfident or insufficiently confident in specifying the prior decision chart. For example, a user with little experience with a process might be tempted to place 95% of the probability into one exposure category, whereas another with considerable experience might be reticent about increasing the probability of one category at the expense of others, resulting in a nearly flat or uniform prior decision chart. Neither accurately expressed their professional judgment.

Prior Distributions

Any of the following can provide information for devising a prior distribution: analysis of past datasets; physical/chemical modeling; experience with similar SEGs or processes and chemicals; and personal, corporate, or trade organization experience. One of the methodological issues to be addressed is the development of guidelines and procedures for devising a defensible informative prior decision distribution. Some basic guidelines are obvious. If we assume complete ignorance of the situation, then by default we have a uniform, noninformative, flat prior where the probability of each exposure category is equal. If, however, we select an initial exposure rating then we are saying that the probability of the true exposure profile being in that exposure category is greater than the probability of any other. It also follows that the probabilities of the remaining categories should be greatest for those close to the primary category, and taper off with distance. In those instances where the initial rating category is at one extreme one would expect that the prior distribution probability of the opposite rating zone to be the least and very low.

We have considered and experimented with three approaches for describing the prior decision distribution:

- Categorical: the prior probability is identical throughout the range of each exposure category
- Univariate, continuous distributions: e.g., rectangular, triangular, and lognormal distributions of 95th percentiles
- Bivariate, continuous distributions: e.g., rectangular and triangular distributions of G and D; lognormal distribution for G and a skewed distribution for D.

It is our current view that the categorical prior is the easiest to work with and use. While the univariate and bivariate (prior) functions are perhaps more mathematically rigorous, we believe that few IHs will have sufficient information from which to devise such priors.

Using the AIHA Exposure Assessment Model to Generate Prior Decision Distributions

The AIHA exposure assessment model⁽¹⁾ encourages IHs to prioritize their exposure assessments by assigning an Initial Rating (i.e., pick a likely exposure category, see Table I), as well as a Certainty Level (high, medium, low) to each SEG.

This initial rating can be based on professional judgment, past data, modeling, etc. (Note that the initial rating and certainty factor are then used to determine whether immediate exposure controls are necessary and to prioritize SEGs for follow-up quantitative exposure assessments.) Because of its simplicity and intuitive nature, the AIHA categorization scheme appeals to us for use in setting or establishing a prior decision distribution. A project is under way to explore the use of the AIHA Initial Rating and Certainty Level concepts for developing prior decision distributions. This work will be reported separately.

Parameter Space

Mathematically, the parameters of a lognormal distribution, the G and D, extend from minimum values of near zero for G and near 1 for D to infinity. Due to physical constraints, these values will be bounded at both ends. (Physical constraints are related to minimum and maximum probable ventilation and generation rates, saturation vapor pressures for liquids, maximum cloud densities for aerosols, and so on.) In reality, sample D values above 4 and G values more than several times the exposure limit are not commonly observed for general industry operations. Because the user must specify minimum and maximum G and D values over which to integrate Eqs. 4 and 5, the minimum and maximum values specified should reflect what we know or can predict regarding the physical boundaries of the G and D plane for the process under consideration.

The results will differ whenever the parameter space is changed. Generally, if a D_{\max} is somewhat larger than any previously observed sample estimate or expected value, the results will be conservative; that is, the decision probabilities will be slightly shifted in favor of the higher exposure categories. In this article we used a D_{\max} of 4, but for stable manufacturing processes a maximum value of 2.5 or 3 might be appropriate. For D_{\min} we used 1.05, because values approaching 1 are unlikely. Again, experience might suggest a larger value.

Similarly, for G_{\min} our recommended default setting is 1/200th the cutoff for the OEL. For G_{\max} our recommended default setting is $5 \cdot \text{OEL}$. These defaults can of course be modified to fit a given situation. For example, say modeling calculations strongly suggest that, given a maximum generation rate, a minimum ventilation rate, and a conservative uniform mixing uncertainty factor, the maximum concentration will not exceed twice the limit. In this case, G_{\max} could be set to this maximum concentration, or even half this value. The effect of modifying the boundaries of the parameter space from the default values is to tailor the BDA calculations to a specific situation resulting, hopefully, in an enhanced analysis and a final decision that is correct. Strictly speaking, constraining the analysis to a specific parameter space is in itself a form of informative prior, but within this parameter space the prior can be noninformative (i.e., uniform, flat) or informative.

ISSUES AND FUTURE WORK

Inconsistent Prior and Likelihood Distributions

Bayesian analysis works best when there is reasonable consistency between the prior distribution and the likelihood. In this situation, the prior reinforces and sharpens our interpretation of the likelihood decision distribution. If the prior and likelihood decision distributions are considerably different, then there is an indication that either our prior was poorly devised or the process has changed significantly since it was last evaluated. (For example, say the following measurements were collected sequentially: $x = \{0.20, 0.05, 0.10, 0.001, 0.002\}$ ppm. The last two are clearly inconsistent with the first three. The process does not appear to be stationary and the data should be critiqued for differences in how the job or task is performed, etc.) There is little in the Bayesian literature on the statistical treatment of an inconsistent prior and likelihood decision distributions. One of our goals is to devise rules of thumb, guidelines, and perhaps statistical tests to be used whenever an inconsistent prior or likelihood distribution is encountered.

Censored Datasets

Censored datasets, or datasets that contain one or more measurements reported as less than the limit of detection, can be difficult to analyze and interpret. There are several methods for dealing with censored datasets but all involve the estimation of the distribution parameters, which for IHs would be geometric mean and geometric standard deviation.^(11,12) The BDA method can be adapted to handle censored datasets, which will be the subject of a future paper.

Use of Exposure Modeling to Generate a Prior Decision Distribution

It is our view that BDA could be used to help bridge the gap between exposure modeling and decision making. Exposure modeling could be used to help develop a prior decision distribution, which would later be used in the BDA model to help interpret the data. Here, there seems to be a prime opportunity for creative thinking regarding how best to integrate exposure modeling and BDA.

Repeat Measurements per Worker

Inspection of exposure datasets often reveals that one or more workers were sampled two or more times. This would suggest that some form of a components-of-variance analysis might be in order. However, in our opinion a dataset consisting of repeat measurements should not present a concern if the decision results in a Category 1 or 2 exposure rating. If the decision is that exposures are just controlled (i.e., a Category 3 exposure), then certainly the factors that may affect worker-to-worker differences in exposure should be evaluated. Readers are referred to the discussion on "critical exposure groups" in Mulhausen and Damiano.⁽¹⁾ In a future publication we hope to extend the BDA model presented here to the analysis of datasets with repeat measurements.

Application to Control Banding

Hewett et al.⁽¹³⁾ suggested that Bayesian Decision Analysis could be used by employers to verify that actual exposures at a specific site are consistent with the target control band (see Russell et al.⁽¹⁴⁾ for an introduction to the European version of control banding). The initial control banding assessment could be adapted to represent a prior decision distribution, which would then be combined with a number of worker or process exposure measurements to determine the posterior probability that exposures are controlled relative to the target control band. (The category cutoffs would be fixed at 0.01, 0.1, 1.0, and 10 mg/m³ for aerosols and 0.5, 5, 50, and 500 ppm for vapors.) The combination of control banding and actual data, using the Bayesian framework introduced here, has the potential to permit efficient and effective decisions regarding the actual control of exposures.

CONCLUSIONS

The advantages of Bayesian decision analysis include: (a) decision probabilities are easily understood by risk managers and employees, (b) prior data, professional judgment, or modeling information are objectively incorporated into the decision-making process, (c) decisions can be made with greater certainty, (d) the probable values for the true geometric mean and geometric standard deviation can be constrained to a plausible parameter space, and (e) fewer measurements are necessary whenever the prior distribution is well defined and the process is fairly stable. Furthermore, Bayesian decision analysis provides an obvious feedback mechanism that can be used by an industrial hygienist to improve professional judgment.

REFERENCES

1. **Mulhausen, J., and J. Damiano (eds.):** *A Strategy for Assessing and Managing Occupational Exposures*, 2nd Edition. Fairfax, Va.: American Industrial Hygiene Association, 1998.
2. **Hawkins, N.C., S.K. Norwood, and J.C. Rock (eds.):** *A Strategy for Occupational Exposure Assessment*. Fairfax, Va.: American Industrial Hygiene Association, 1991.
3. **Hagan, A.:** *Kendall's Advanced Theory of Statistics, Volume 2B—Bayesian Inference*, 1st Edition. London: Arnold, 1994.
4. **Silvia, D.S.:** *Data Analysis—A Bayesian Tutorial*, 1st Edition. New York: Oxford University Press, 1996.
5. **Gelman, A., J.B. Carlin, H.S. Stern, and D.B. Rubin:** *Bayesian Data Analysis*, 2nd Edition. Boca Raton, Fla.: Chapman & Hall/CRC, 2004.
6. **Jayjock, M.A.:** Uncertainty analysis in the estimation of exposure. *Am. Ind. Hyg. Assoc. J.* 58:380–382 (1997).
7. **American Industrial Hygiene Association (AIHA):** *Mathematical Models for Estimating Occupational Exposure to Chemicals*. AIHA Press, Fairfax, Va.: 2000.
8. “Respiratory Protection” *Code of Federal Regulations*, Title 29, Part 1910.134 (d)(1)(iii), 2004.
9. **Hewett, P.:** “Improving the Effectiveness and Efficiency of the IH Decision-Making Process.” Poster presented at the 2003 American Industrial Hygiene Conference and Exposition (AIHce), Dallas, Texas. In *AIHce 2003 Abstracts*, Abstract #278, 2003.

10. **Hewett, P., P. Logan, G. Ramachandran, and J. Mulhausen:** “Combining Professional Judgment and Exposure Data—Bayesian Decision Analysis.” Presented at the British Occupational Hygiene Society Occupational Hygiene 2004 Conference and Exhibition, Stratford-upon-Avon, England, 2004.
11. **Hornung, R.W., Reed, L.D.:** Estimation of average concentration in the presence of nondetectable values. *Appl. Occup. Environ. Hyg.* 5:46–51 (1990).
12. **Finkelstein, M.M., Verma, D.K.:** Exposure estimation in the presence of nondetectable values: Another look. *AIHAJ* 62(2):195–198 (2001); Letters to the Editor *AIHAJ* 63:4 (2002).
13. **Hewett, P., Mulhausen, J., Logan, P.:** “A Proposal for the Validation of Control Banding Using Bayesian Decision Analysis Techniques.” Presented at the 2nd International Control Banding Workshop: Validation and Effectiveness of Control Banding, Cincinnati, Ohio, 2004.
14. **Russell, R.M., Maidment, S.C., Brooke, I., Topping, M.D.:** An introduction to a UK scheme to help small firms control health risks from chemicals. *Ann. Occup. Hyg.* 42:367–376 (1998).

APPENDIX I

WinBUGS Code

Below is the WinBUGS code used to generate a distribution of 95th percentiles. This distribution is then exported to a spreadsheet and the fraction of values in each exposure category is then determined.

```
model {
# This line specifies the model used in the iterations. We are
sampling from a lognormal distribution with mean (mu) and
precision (tausq)
# The precision is related to the variance (sigmasquared) by
the following equation—tausq = 1/ sigmasquared
for (i in 1:N) {
Y[i] ~ dlnorm(mu,tausq)
}
# Set a uniform prior on the mean of the lognormal distribution
between bounds of ln(0.05) = -0.3 and ln(500) = 6.2
mu ~ dunif(-3.0,6.2)
# Set a uniform prior on the standard deviation of the lognormal
distribution between bounds of 0.05 (GSD=1.05) and 1.39
(GSD=4)
sigma ~ dunif(0.05, 1.39)
# WinBUGS specifies tausq (tau squared) as the precision so
we must convert sigma to tausq
tausq <- 1/pow(sigma, 2)
# The following 3 lines are used to calculate the point estimate
of the 95th percentile for each iteration
Geom.mean <- exp(mu)
Geo.sigma <- exp(sigma)
Xpe <- Geom.mean*(pow(Geo.sigma,1.645))
}
#Inits—These are the initial values of the simulation
list(mu=0.5, sigma=1.0)

#Data—Here are the data points for the model
list(N=3, Y=c(0.20, 0.05, 0.10))
```